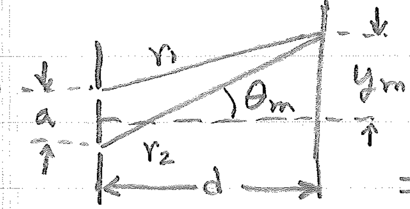
Physics 302 Photonics

HW-7 Chapter 9 Solutions

(9.9)



$r_2 - r_1 = a \sin\theta_m = m\lambda \cong a\theta_m$ for small angles

$\Rightarrow \theta_m = m\lambda/a$

$\tan\theta_m = y_m/d \cong \theta_m$

So $\Delta\theta = \theta_{m+1} - \theta_m = \frac{(m+1)\lambda}{a} - \frac{m\lambda}{a} = \frac{\lambda}{a}$

Also $\Delta\theta = \frac{y_{m+1}}{d} - \frac{y_m}{d} = \frac{\Delta y}{d}$

So $\frac{\Delta y}{d} = \frac{\lambda}{a} \Rightarrow d = \frac{a\,\Delta y}{\lambda} = \frac{(0.10\text{ mm})(10\text{ mm})}{487.99\text{ nm}}$

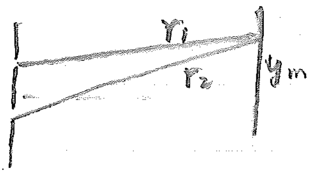
$$\boxed{d = 2.05\text{ m}}$$

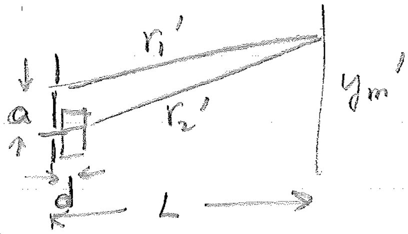
(9.10) From prob 9.9, $\theta_m = m\lambda/a$

Given: $\theta_1(\text{Red}) = \theta_2(\text{Violet})$

$\Rightarrow \frac{1\,\lambda_{red}}{a} = \frac{2\,\lambda_{violet}}{a}$

or $\lambda_{violet} = \frac{\lambda_{red}}{2} = 395\text{ nm}$

(9.12) 

vs 

without glass: $OPL_1 = r_1$, $OPL_2 = r_2$ $\Big\}$ $r_2 - r_1 = m\lambda = a\,\theta_m$

with glass: $OPL_1' = r_1'$

$OPL_2' = (r_2' - d)\,1 + d \cdot n$

Now $OPL_2' - OPL_1' = (r_2' - r_1') + d(n-1) = m\lambda = a\,\theta_m'$

Combining: $r_2 - \lambda_1 = (r_2' - r_1') + d(n-1)$

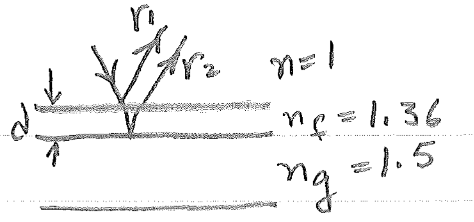
or $a\,\theta_m = a\,\theta_m' + d(n-1)$

But $\theta_m = y_m/L$ and $\theta_m' = y_m'/L$

So $\frac{a}{L} y_m = \frac{a}{L} y_m' + d(n-1)$

$$\Rightarrow \boxed{y_m - y_m' = \frac{dL}{a}(n-1)}$$

(9.27)



Since both reflections are "external", both have a phase change of π.

["External reflection" means $n_i < n_t$]

If we assume normal incidence, the path difference is $r_2 - r_1 = 2d$. Since this path difference gives constructive interference for green light, $\lambda = 500\text{ nm}$, the phase difference will be $m\,2\pi$, $m = 1, 2, 3, \ldots$, ie

$$2\pi\left(\frac{r_2 - r_1}{\lambda_f}\right) = m\,2\pi$$

or $\Delta r = m\lambda_f = m\,\lambda_0/n_f$

But $d_m = \dfrac{\Delta r}{2} = m\dfrac{\lambda_0}{2n_f} = m\,(184\text{ nm})$

The thinnest film giving constructive interf. for green light is

$d_1 = 184\text{ nm}$

(9.36) Change in phase $= \Delta\delta = k$ (change in path length)

$$= k(2\Delta x)$$

But if N fringes pass, $\Rightarrow \Delta\delta = N \cdot 2\pi$

$$\Rightarrow \Delta x = \frac{N \cdot 2\pi}{2k} = \frac{N \cdot 2\pi}{2 \cdot (2\pi/\lambda)} = \frac{N\lambda}{2}$$

$$= \frac{(1000 \text{ fringes})(500 \text{nm})}{2}$$

$$\boxed{\Delta x = 2.5 \times 10^{5} \text{ nm} = 0.25 \text{ mm}}$$